What is the Vector Potential, and how do we use it?

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VPPEM uses the magnetic vector potential field as a spatially resolved reference for image formation. A strong solenoid (superconducting) magnet is used to create the vector potential field. This application of the vector potential to microscopy is unique, and it is useful to give some background to the properties of the vector potential field.

The vector potential field was originally conceived by Maxwell, and unifies the electric and magnetic fields. The electric field E, and magnetic field **B**, are given by the differentials of the scalar potential φ, and the vector potential **A**.

The electric and magnetic fields in potential form are:

$$E = -\nabla . \varphi - \frac{\partial \mathbf{A}}{\partial t} \quad (1)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (2)$$

The vector potential field is often considered to be only a mathematical convenience, but for our purposes the field can be thought of as a consequence of the momentum of the conduction electron wave functions, and is very real. That is to say, the wave function of a moving electron carries momentum, and that momentum is seen as a momentum field at a distance r from the electron. The vector potential from a small element of current density j in a volume V is:

$$\mathbf{A}(\mathbf{p_1}, t) = \frac{\mu_0}{4\pi} \int_{V_2} \frac{\mathbf{j}(\mathbf{p_2}, t_r)}{r_{12}} \mathrm{d}V \quad (3)$$

Integrating this expression along a circuit gives the direction and magnitude of **A**. In the case of radially symmetric current carrying solenoid, this momentum field is radially symmetric, zero on the axis, and increases linearly with radius out to the solenoid. For a radially symmetric solenoid vector potential field, the resultant magnetic field **B** is a constant.

The vector potential field from a solenoid is conceptually illustrated in Figure 1. The vector potential field **A** rotates clockwise around the axis of the solenoid in the direction of the arrows. The vector potential field magnetic field **B** is perpendicular into the figure along the axis of the solenoid. The direction and magnitude of the vector potential field can be

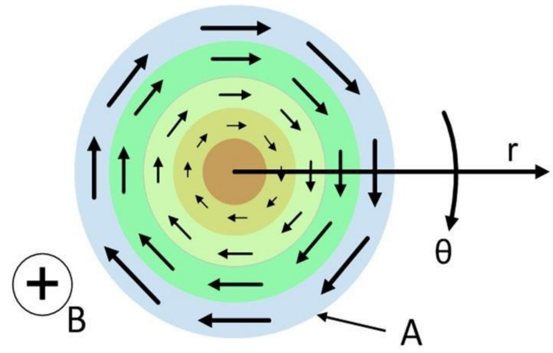

Figure 1. Vector potential as a spatial reference

seen to form a two dimensional spatial reference in angle θ, and radius r. It is this spatial reference that we use by placing a sample in center of the field so that photoelectrons emitted from the sample have a momentum component due to the field. This momentum component is dependent on where on the sample the electrons are emitted.

The vector potential extends out of Figure 1 along the axis of the solenoid. Conceptually the contours of equal vector potential are long cylinders. For a solenoid such as a 20 cm long superconducting magnet, the reference field is effectively constant over several centimeters at the center of the coil. Therefore, the microscope has a very large working 'depth of focus', and can image structures with extremely high aspect ratios.

The vector potential field is a momentum field with dimensions of momentum per unit charge. If an electron leaves the field suddenly, by definition, it gains momentum due to its charge:

$$\boldsymbol{p} = -\boldsymbol{eA} \quad (3)$$

Where e is the charge on the electron, and the momentum due to **A** is **p**. Equivalently, we could consider the gain in momentum is due to integrating the effect of the electric field E that the electron 'sees' due to the change in **A** over the time the electron takes to exit the field (from Equation 1).

The electron optical properties of a vector potential microscope can be illustrated by simulating the exit of electron trajectories from a simple magnetic circuit.

In Figure 2 a rotationally symmetric magnetic field is modeled using a permanent magnet in a soft iron yoke with a tapered aperture for the exit of electrons from the field.

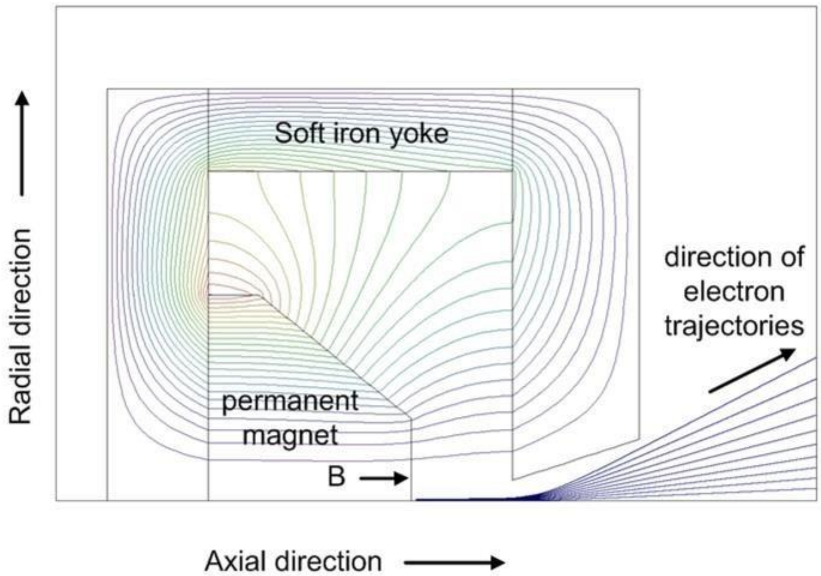

Figure 2 Deflection of electron trajectories through a magnetic aperture.

The vector potential near the center of the pole piece will be cylindrical as shown in Figure 2 because it will be a relatively constant magnetic field near the pole piece. Electron trajectories begin at equally spaced radial points just off the axis near the pole piece. The electron trajectories initially travel along parallel to the optical axis, and are constrained into a narrow beam by the magnetic field lines near the axis. There is a small magnification of the radial distances as the magnetic field weakens moving away from the pole piece.

When the electrons exit the field through the aperture, they leave the field lines, and become deflected away from the axis by the change in the vector potential. The momentum in the vector potential is conserved, and produces an angular distribution of the electrons dependent on their initial radial distribution in the vector potential field.

Only the radial distances are plotted in Figure 2, the electron trajectories are actually rotated out of the plane of the diagram. The deflection angles agree with Equation 3 to a few percent, and are slightly dependent on the details of the aperture.

Figure 2 illustrates several points. The trajectories are relatively insensitive to the starting point near the pole piece, so that the effective focal depth is large. The electrons are initially constrained in a beam within the magnetic field before exiting the aperture which gives a long working distance. This long working distance means there is a volume around the sample that is accessible for other instrumentation, evaporators, and an environmental chamber. The long working distance also allows the illumination to come in at an angle close to the optical axis. The simulation shows the deflection angles are proportional to the radial positions at the origin, for small and moderate angles, which implies that interpretation of the image is straightforward.

A solenoidal vector potential field is gauge invariant. In practical terms, this means that the center of the field, and thus, the microscope axis can be arbitrarily defined.

IN CONCLUSION the vector potential gives us a spatial reference that leads to an angular image. This angular image is straightforward to energy analyze, and project on an image plane.

Note that in VPPEM the object and image planes are not at conjugate planes. There is no real 'focus' but a large depth of field.